

Solving Polynomials Homework

1.) Let $p(x) = x^3 - x^4 + 8x^2 - 9x + 30$. Evaluate $p(-2)$. What does the solution tell you about $p(x)$? $(-2)^3 - (-2)^4 + 8(-2)^2 - 9(-2) + 30 = \boxed{88}$

This isn't a factor, but when plug in -2 you get 88 which is a remainder.

2.) Consider the polynomial function: $p(x) = x^4 - 3x^3 + ax^2 - 6x + 14$, where a is an unknown

$$2^4 - 3(2)^3 + a(2)^2 - 6(2) + 14 = 0$$

$$16 - 24 + 4a + 2 - 12 + 14 = -8 + 4a + 2 = -6 + 4a = 0$$

$$\frac{4a}{4} = \frac{6}{4} = \boxed{\frac{3}{2}}$$

* Should always get zeros because they're roots!
For #3-6, find all the solutions (real and complex) for the following polynomials. If a solution has a multiplicity, make sure to state it. zeros factors!

3.) $f(x) = x^4 - 7x^2 + 12$

$$\boxed{x=2}$$

$$\boxed{x=-2}$$

$$\begin{array}{r|rrrrrr} 2 & 1 & 0 & -7 & 0 & 12 \\ & \downarrow & & & & \\ \hline & 1 & 2 & -3 & -6 & 0 \end{array}$$

$$1x^3 + 2x^2 - 3x - 6$$

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -3 & -6 \\ & \downarrow & & & \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$$x^2 - 3 = 0$$

$$\sqrt{x^2} = \sqrt{3}$$

$$\boxed{x = \pm\sqrt{3}}$$

5.) $f(x) = x^3 - x + 6$
 $x^3 + 0x^2 - x + 6$

$$\boxed{x=-2}$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -1 & 6 \\ & \downarrow & & & \\ \hline & 1 & -2 & 3 & 0 \end{array}$$

$$x^2 - 2x + 3 = 0$$

$$\frac{2 \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)} = \frac{2 \pm \sqrt{4-12}}{2} = \frac{2 \pm \sqrt{-8}}{2} = \frac{2 \pm 2i\sqrt{2}}{2} =$$

$$\boxed{1 \pm i\sqrt{2}}$$

4.) $f(x) = x^4 - x^3 + 25x^2 - 25x$

$$\boxed{x=0, 1}$$

$$\begin{array}{r|rrrrr} 0 & 1 & -1 & 25 & -25 & 0 \\ & \downarrow & & & & \\ \hline & 1 & -1 & 25 & -25 & 0 \end{array}$$

$$(x^3 - x^2) + (25x - 25)$$

$$x^2(x-1) + 25(x-1)$$

$$(x^2 + 25)(x-1)$$

6.) $f(x) = x^5 + 4x^4 - 41x^2 + 36$

$$\boxed{x=-2, -1, 1, 2}$$

$$\begin{array}{r|rrrrrrr} -2 & 1 & 0 & 4 & 0 & -41 & 0 & 36 \\ & \downarrow & & & & & & \\ \hline & 1 & -2 & 8 & -16 & -9 & 18 & 0 \end{array}$$

→

$$x^5 - 2x^4 + 8x^3 - 16x^2 - 9x + 18$$

$$\begin{array}{r|rrrrrr} -1 & 1 & -2 & 8 & -16 & -9 & 18 \\ & \downarrow & -1 & 3 & -11 & 27 & -18 \\ \hline & 1 & -3 & 11 & -27 & 18 & 0 \end{array}$$

$$x^4 - 3x^3 + 11x^2 - 27x + 18$$

$$\begin{array}{r|rrrrr} 1 & 1 & -3 & 11 & -27 & 18 \\ & \downarrow & 1 & -2 & 9 & -18 \\ \hline & 1 & -2 & 9 & -18 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & 9 & -18 \\ & \downarrow & 2 & 0 & 18 \\ \hline & 1 & 0 & 9 & 0 \end{array}$$

$$x^2 + 9 = 0$$

$$\sqrt{x^2} = \sqrt{-9}$$

$$x = \pm 3i$$

#goal: always get it to a quadratic (should never have a cubic)

#3

$$(x-2)(x-2)(x+3)(x+3)(x+3)(x-4)$$

$$(x-2)^2(x+3)^3(x-4)$$

degree = 6