

State the number of complex roots, the possible number of real and imaginary roots, the possible number of positive and negative roots, and the possible rational roots for each equation. Then find all roots.

1) $x^4 - 5x^2 - 36 = 0$ **Treat like a Quad.**
 $(x^2 - 9)(x^2 + 4) = 0$

$x^2 - 9 = 0$ $x^2 + 4 = 0$
 $\sqrt{x^2} = \sqrt{9}$ $\sqrt{x^2} = \sqrt{-4}$
 $x = \pm 3$ $x = \pm 2i$

$\{ \pm 3, \pm 2i \}$

3) $x^3 - 2x^2 + 3x - 6 = 0$
 $x^2(x-2) | 3(x-2)$

$(x^2 + 3)(x - 2) = 0$
 $x^2 + 3 = 0$ $x - 2 = 0$
 $\sqrt{x^2} = \sqrt{-3}$ $x = 2$

$x = \pm i\sqrt{3}$

$\{ 2, \pm i\sqrt{3} \}$

5) $x^4 + 6x^2 + 8 = 0$

$(x^2 + 4)(x^2 + 2) = 0$
 $x^2 + 4 = 0$ $x^2 + 2 = 0$
 $\sqrt{x^2} = \sqrt{-4}$ $\sqrt{x^2} = \sqrt{-2}$
 $x = \pm 2i$ $x = \pm i\sqrt{2}$

$\{ \pm 2i, \pm i\sqrt{2} \}$

7) $x^3 - 1 = 0$ SOAP!

$(x-1)(x^2 + x + 1) = 0$
 $x - 1 = 0$ $x^2 + x + 1 = 0$
 $x = 1$

$$\frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

$$\frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$$

$\{ 1, \frac{-1 \pm i\sqrt{3}}{2} \}$

2) $x^3 + 3x^2 - 14x - 20 = 0$
 $x^2(x+3) | -2(7x+5)$

$$\begin{array}{r} -5 \\ \underline{1^3 - 2^2 - 4^1 } \\ 1^2 - 2^1 - 4^0 \end{array}$$

$x^2 - 2x - 4 = 0$

$(-\frac{2}{2})^2 = 1$ $x^2 - 2x + 1 = 4 + 1$
 $(x-1)^2 = 5$

4) $x^4 - 14x^2 + 45 = 0$
 $(x^2 - 5)(x^2 - 4) = 0$

$x^2 - 5 = 0$ $x^2 - 4 = 0$
 $\sqrt{x^2} = \sqrt{5}$ $\sqrt{x^2} = \sqrt{4}$
 $x = \pm \sqrt{5}$ $x = \pm 2$

$\{ \pm \sqrt{5}, \pm 2 \}$

**can't factor - graph!*

Rational. Since the other 2 are irrational then you must find their exact solutions (no decimals)

$\sqrt{(x-1)^2} = \sqrt{5}$
 $x - 1 = \pm \sqrt{5}$
 $x = 1 \pm \sqrt{5}$
 $\{ -5, 1 \pm \sqrt{5} \}$

6) $x^4 + 3x^2 - 18 = 0$

$(x^2 + 6)(x^2 - 3) = 0$
 $x^2 + 6 = 0$ $x^2 - 3 = 0$
 $\sqrt{x^2} = \sqrt{-6}$ $\sqrt{x^2} = \sqrt{3}$
 $x = \pm i\sqrt{6}$ $x = \pm \sqrt{3}$

$\{ \pm i\sqrt{6}, \pm \sqrt{3} \}$

8) $x^3 + 3x^2 - x - 3 = 0$

$x^2(x+3) | -1(x+3)$
 $(x^2 - 1)(x + 3) = 0$
 $x^2 - 1 = 0$ $x + 3 = 0$
 $\sqrt{x^2} = \sqrt{1}$ $x = -3$
 $x = \pm 1$

$\{ -3, -1, 1 \}$

$$9) x^3 - 2x^2 - 3x + 6 = 0$$

$$x^2(x-2) - 3(x-2)$$

$$(x^2-3)(x-2) = 0$$

$$x^2-3=0 \quad x-2=0$$

$$\sqrt{x^2} = \sqrt{3} \quad x=2$$

$$x = \pm\sqrt{3}$$

$$\boxed{\{\pm\sqrt{3}, 2\}}$$

$$10) x^6 - 2x^4 - 4x^2 + 8 = 0$$

$$x^4(x^2-2) - 4(x^2-2)$$

$$(x^4-4)(x^2-2) = 0$$

Keep Factoring!

$$(x^2+2)(x^2-2)(x^2-2) = 0$$

$$x^2+2=0 \quad x^2-2=0 \quad x^2-2=0$$

$$\sqrt{x^2} = \sqrt{-2} \quad \sqrt{x^2} = \sqrt{2} \quad \sqrt{x^2} = \sqrt{2}$$

$$x = \pm i\sqrt{2} \quad x = \pm\sqrt{2} \quad x = \pm\sqrt{2}$$

$$\boxed{\{\pm i\sqrt{2}, \pm\sqrt{2} \text{ mult } 2\}}$$

Graph !!

$$11) x^5 + 2x^4 + 11x^3 + 22x^2 + 24x + 48 = 0$$

$$x = -2$$

$$-2 \left| \begin{array}{cccccc} 5 & 4 & 3 & 2 & 1 & 0 \\ 1 & 2 & 11 & 22 & 24 & 48 \\ \downarrow & -2 & 0 & -22 & 0 & -48 \\ 1 & 0 & 11 & 0 & 24 & 0 \end{array} \right.$$

$$x^4 + 11x^2 + 24 = 0 \text{ FACTOR!}$$

$$(x^2+8)(x^2+3) = 0$$

$$x^2+8=0 \quad x^2+3=0$$

$$\sqrt{x^2} = \sqrt{-8} \quad \sqrt{x^2} = \sqrt{-3}$$

$$x = \pm 2i\sqrt{2} \quad x = \pm i\sqrt{3}$$

$$\boxed{\{-2, \pm 2i\sqrt{2}, \pm i\sqrt{3}\}}$$

$$12) x^6 + 5x^4 - 4x^2 - 20 = 0$$

$$x^4(x^2+5) - 4(x^2+5)$$

$$(x^4-4)(x^2+5) = 0$$

$$(x^2+2)(x^2-2)(x^2+5) = 0$$

$$x^2+2=0 \quad x^2-2=0 \quad x^2+5=0$$

$$\sqrt{x^2} = \sqrt{-2} \quad \sqrt{x^2} = \sqrt{2} \quad \sqrt{x^2} = \sqrt{5}$$

$$x = \pm i\sqrt{2} \quad x = \pm\sqrt{2} \quad x = \pm i\sqrt{5}$$

$$\boxed{\{\pm i\sqrt{2}, \pm\sqrt{2}, \pm i\sqrt{5}\}}$$

$$13) x^6 - x^4 - x^2 + 1 = 0$$

$$x^4(x^2-1) - 1(x^2-1)$$

$$(x^4-1)(x^2-1) = 0$$

$$(x^2+1)(x^2-1)(x^2-1)$$

$$x^2+1=0 \quad x^2-1=0 \quad x^2-1=0$$

$$\sqrt{x^2} = \sqrt{-1} \quad \sqrt{x^2} = \sqrt{1} \quad \sqrt{x^2} = \sqrt{1}$$

$$x = \pm i \quad x = \pm 1 \quad x = \pm 1$$

$$\boxed{\{\pm i, \pm 1 \text{ mult } 2\}}$$

$$14) x^8 - 26x^4 + 25 = 0$$

$$(x^4-25)(x^4-1)$$

$$(x^2-5)(x^2+5)(x^2-1)(x^2+1)$$

$$x^2-5=0 \quad x^2+5=0 \quad x^2-1=0 \quad x^2+1=0$$

$$\sqrt{x^2} = \sqrt{5} \quad \sqrt{x^2} = \sqrt{-5} \quad \sqrt{x^2} = \sqrt{1} \quad \sqrt{x^2} = \sqrt{-1}$$

$$x = \pm\sqrt{5} \quad x = \pm i\sqrt{5} \quad x = \pm 1 \quad x = \pm i$$

$$\boxed{\{\pm\sqrt{5}, \pm i\sqrt{5}, \pm 1, \pm i\}}$$