Chapter 8 Review

Write an equation for the translation of $y = \dfrac{2}{x}$ with the given asymptotes.

1. $x = 1, y = -1$ $y = \dfrac{2}{x-1} - 1$

2. $x = 5, y = \dfrac{1}{2}$ $y = \dfrac{2}{x-5} + \dfrac{1}{2}$

For each rational function, identify any holes or horizontal or vertical asymptotes of its graph.

3. $y = \dfrac{x}{x-3}$ no holes
VA $x = 3$
HA $y = 1$

4. $y = \dfrac{-2(x-8)}{8-x} = \dfrac{-2(x-8)}{-(x-8)}$ $y = 2$ hole at $x = 8$ no V.A.   HA $y = 2$

5. $y = \dfrac{x+3}{x^2 - 4x - 21}$ $y = \dfrac{1}{x-7}$
$(x+3)(x-7)$
hole @ $x = -3$ HA $y = 0$
VA $x = 7$

6. $y = \dfrac{1}{x+4} - 3$ no holes
VA $x = -4$
HA $y = -3$

Sketch the graph of each rational function.

7. $y = \dfrac{x}{x(x-2)}$ $y = \dfrac{1}{x-2}$

8. $y = \dfrac{1}{x+4} - 3$

Simplify each rational expression. State any restrictions on the variable.

9. $\dfrac{3x^2 - 12}{x^2 - x - 6}$ $\dfrac{3(x^2 - 4)}{(x+2)(x-3)}$
$\dfrac{3(x+2)(x-2)}{(x+2)(x-3)} \to \dfrac{3(x-2)}{x-3}$ $x \neq -2, 3$

10. $\dfrac{2x^2 - x}{4x^2 - 4x + 1} \div \dfrac{x}{8x - 4}$
$\dfrac{x(2x-1)}{(2x-1)(2x-1)} \cdot \dfrac{4(2x-1)}{x} = 4$ $x \neq \dfrac{1}{2}, 0$

Find the least common multiple of each pair of polynomials.

11. $x^2 - 16$ and $5x + 20$
$(x-4)(x+4)$ $5(x+4)$
$5(x-4)(x+4)$

12. $7(x-2)(x+5)$ and $2(x+5)^2$
$14(x-2)(x+5)^2$

Simplify each sum or difference.

13. $\dfrac{2}{x+5} + \dfrac{x}{x-5}$
$\dfrac{2(x-5) + x(x+5)}{(x+5)(x-5)} = \dfrac{2x - 10 + x^2 + 5x}{(x+5)(x-5)} = \dfrac{x^2 + 7x - 10}{(x+5)(x-5)}$

14. $\dfrac{3x}{x^2 - 4} - \dfrac{1}{x^2}$
$(x-2)(x+2)$
$\dfrac{3x \cdot x^2 - (x^2 - 4)}{x^2(x-2)(x+2)} = \dfrac{3x^3 - x^2 + 4}{x^2(x-2)(x+2)}$ $x \neq 0, \pm 2$

Simplify each complex fraction.

15. $\dfrac{1 + \dfrac{2}{3}}{}$

16. $\dfrac{1 + \dfrac{1}{x}}{}$

Solve each equation. Check each solution.

17. $\dfrac{x}{3} + \dfrac{x}{2} = 10$ LCD: 6

$2x + 3x = 60$

$5x = 60$

$\boxed{x = 12}$

18. $\dfrac{y-3}{5} = \dfrac{y+1}{7}$ LCD: 35

$7(y-3) = 5(y+1)$

$7y - 21 = 5y + 5$

$2y = 26 \quad \boxed{y = 13}$

19. $\dfrac{x}{2} = 2x - 3$ LCD: 2

$x = 2(2x-3)$

$x = 4x - 6$

$-3x = -6 \quad \boxed{x = +2}$

20. $-\dfrac{x}{4} = \dfrac{2x}{3}$ LCD: 12

$-3x = 8x$

$\boxed{x = 0}$

21. $\dfrac{1}{x} - \dfrac{1}{6} = \dfrac{4}{3x^2}$

$6x - x^2 = 8$

$x^2 - 6x + 8 = 0$

$(x-2)(x-4) = 0 \quad \boxed{x = 2, 4}$

22. $\dfrac{2x-4}{x-5} = 0$

$2x - 4 = 0$

$2x = 4$

$\boxed{x = 2}$

23. Chad can paint a room in 2 h. Cassie can paint the room in 3 h. How long would it take them to paint the room working together?

$\dfrac{x}{2} + \dfrac{x}{3} = 1$

$x\left(\dfrac{1}{2} + \dfrac{1}{3}\right) = 1$

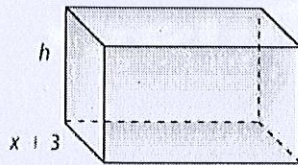$3x + 2x = 6$

$5x = 6$

$\boxed{x = 1.2\ hrs}$

Do you UNDERSTAND?

24. **Reasoning** Is $(3, y)$ on the graph of $y = \dfrac{3}{x-3} + 3$? Justify your answer.

No, the graph is undefined at $x = 3$

25. **Reasoning** Write an expression in simplest form for the height of the rectangular prism shown at the right.

h $V = \dfrac{x^2 + x - 6}{x}$ (base $x + 3$, width $\frac{1}{x}$)

$(h)(x+3)\left(\dfrac{1}{x}\right) = x^2 + x - 6$

$\dfrac{x}{x+3} \cdot \dfrac{h(x+3)}{x} = x^2 + x - 6 \cdot \dfrac{x}{x+3}$
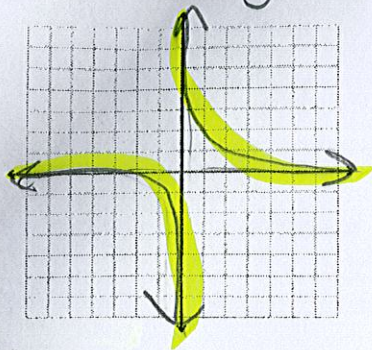
$h = \dfrac{(x+3)(x-2)}{x} \cdot \dfrac{x}{x+3}$
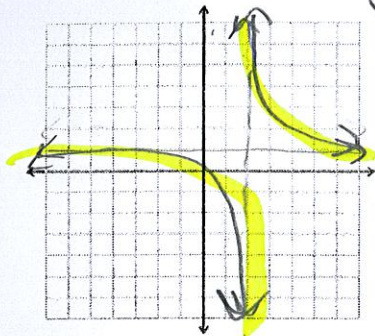
$\boxed{h = x + 2}$

Rational Review – Chapter 8

Graph each function. Identify any vertical asymptotes or holes.
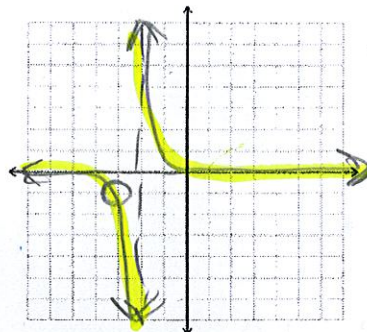
1. $y = \frac{1}{x}$ VA $x=0$ HA $y=0$

2. $y = \frac{5}{x-2} + 1$ VA $x=2$ HA $y=1$

3. $y = \frac{x+3}{x^2+5x+6}$ $\frac{(x+3)}{(x+3)(x+2)}$ $y = \frac{1}{x+2}$

VA $x=-2$ HA $y=0$

Simplify each rational expression. State any restrictions on the variable.

4. $\frac{x^2-1}{x^2+6x+5}$ $\frac{(x+1)(x-1)}{(x+5)(x+1)}$

$\frac{x-1}{x+5}$ $x \neq -5, -1$

5. $\frac{4x}{5y} \cdot \frac{10y^4}{24x^3}$ $\frac{40xy^4}{120x^3y}$

$\frac{y^3}{3x^2}$

6. $\frac{x^2+2x-15}{x-3} \div \frac{x^2-4}{2}$

$\frac{(x+5)(x-3)}{x-3} \cdot \frac{2}{(x+2)(x-2)}$

$\frac{2(x+5)}{(x+2)(x-2)}$ $x \neq \pm 2, 3$

7. $\frac{2}{x+5} + \frac{x}{x-5}$

$\frac{2(x-5) + x(x+5)}{(x+5)(x-5)}$

$\frac{2x-10+x^2+5x}{(x+5)(x-5)}$

$\frac{x^2+7x-10}{(x+5)(x-5)}$ $x \neq \pm 5$

8. $\frac{x^2+3x-10}{x^2-4} - \frac{3}{x+2}$

$\frac{(x+5)(x-2)}{(x+2)(x-2)}$

$\frac{x+5}{x+2} - \frac{3}{x+2}$

$\frac{x+2}{x+2} = 1$ $x \neq \pm 2$

9. $\frac{x+4}{x+4} \cdot \frac{x-4}{x^2-2x-8} - \frac{x-2}{x^2-16}$

$\frac{1}{(x-4)(x+2)}$ $\frac{(x+2)}{(x+4)(x-4)}$

$\frac{x^2-16 - (x^2+4)}{(x+4)(x-4)(x+2)}$

$\frac{-12}{(x+4)(x-4)(x+2)}$ $x \neq \pm 4, -2$

Simplify. State any restrictions on the variables.

10. $\frac{4d^2+8d}{2d}$

$\frac{4d(d+2)}{2d}$

$2(d+2)$

$d \neq 0$

11. $\frac{x^2+9x+18}{x+6}$

$\frac{(x+6)(x+3)}{x+6}$

$x+3$ $x \neq -6$

12. $\frac{x^2-2x-8}{x+3} \div \frac{x-4}{x+3}$

$\frac{(x+2)(x-4)}{x+3} \cdot \frac{x+3}{x-4}$

$x+2$ $x \neq -3, 4$

13. $\frac{3x+1}{x^2-x-6} \div \frac{6x^2+11x+3}{x^2+4x+4}$ $\frac{(x+2)(x+2)}{(3x+1)(2x+3)}$

$(x-3)(x+2)$

$\frac{x+2}{(x-3)(2x+3)}$ $x \neq 3, -\frac{1}{3}, -\frac{3}{2}, -2$

14. $\frac{x^2+3x+2}{x-1} \cdot \frac{1-x}{x+2}$ $\frac{(x+2)(x+1)}{x-1} \cdot \frac{-(x-1)}{x+2}$

$-(x+1)$ $x \neq 1, -2$

15. $\dfrac{2}{2} \dfrac{6x+1}{x+2} + \dfrac{2x-5}{2x+4}$

$2(x+2)$

$\dfrac{12x+2+2x-5}{2(x+2)} = \underline{\dfrac{14x-3}{2(x+2)}}$

$x \neq -2$

16. $\dfrac{x-3}{x^2+3x} + \dfrac{7}{x+3} \times \dfrac{x}{x}$

$x(x+3)$

$\dfrac{x-3+7x}{x(x+3)} = \underline{\dfrac{8x-3}{x(x+3)}}$

17. $\dfrac{3x}{x^2+5x+6} - \dfrac{2x}{x^2+8x+16}$

$(x+2)(x+3) \quad (x+4)(x+4)$

$\dfrac{3x(x^2+8x+16) - 2x(x^2+5x+6)}{(x+2)(x+3)(x+4)^2}$

$3x^3 + 24x^2 + 48x - 2x^3 - 10x^2 - 12x$

$\dfrac{x^3+14x^2+36x}{(x+2)(x+3)(x+4)^2}$

$\underline{\dfrac{x(x^2+14x+36)}{(x+2)(x+3)(x+4)^2}} \qquad x \neq -2, -3, -4$

18. $\dfrac{2x}{x-5} - \dfrac{x}{x+7}$

$\dfrac{2x(x+7) - x(x-5)}{(x-5)(x+7)}$

$\dfrac{2x^2+14x-x^2+5x}{(x-5)(x+7)}$

$\dfrac{x^2+19x}{(x-5)(x+7)} = \underline{\dfrac{x(x+19)}{(x-5)(x+7)}}$

$x \neq 5, -7$

Solve each equation. Check for extraneous solutions.

19. $\dfrac{1}{x}^{\,5x} \dfrac{6}{5x}^{\,5x} + 1^{\,5x}$ LCD: $5x$

$5 = 6 + 5x$

$-1 = 5x$

$\underline{x = -\dfrac{1}{5}}$

20. $\dfrac{1}{6x^2}^{\,6x^2} \dfrac{1}{2x}^{\,6x^2} + \dfrac{7}{6x^2}^{\,6x^2}$ LCD: $6x^2$

$1 = 3x + 7$

$-6 = 3x$

$\underline{x = -2}$

21. $\dfrac{1}{v}^{\,v(v-5)} + \dfrac{3v+12}{v^2-5v}^{\,v(v-5)} = \dfrac{7v-56}{v^2-5v}^{\,v(v-5)}$ LCD: $v(v-5)$

$v-5 + 3v+12 = 7v-56$

$4v+7 = 7v-56$

$63 = 3v$

$\underline{v = 21}$

22. $\dfrac{1}{m^2-m}^{\,m(m-1)} + \dfrac{1}{m}^{\,m(m-1)} = \dfrac{5}{m^2-m}^{\,m(m-1)}$ LCD: $m(m-1)$

$1 + m-1 = 5$

$\underline{m = 5}$

23. $\dfrac{1}{n-8}^{\,n-8} - 1^{\,n-8} = \dfrac{7}{n-8}^{\,n-8}$ LCD: $n-8$

$1 - (n-8) = 7$

$1 - n + 8 = 7$

$-n + 9 = 7$

$-n = -2$

$\underline{n = 2}$

24. $\dfrac{1}{r-2}^{\,(r-5)(r-2)} + \dfrac{1}{r^2-7r+10}^{\,(r-5)(r-2)} = \dfrac{6}{r-2}^{\,(r-5)(r-2)}$ LCD: $(r-5)(r-2)$

$(r-5)(r-2)$

$r-5 + 1 = 6(r-5)$

$r-4 = 6r-30$

$-5r = 26$

$\underline{r = 5.2}$