speed Dating

VII. Exponential & Logarithmic Functions

Solve each equation.

$y = \log_b x$ if and only if $x = b^y$.
Think of $y = \log_b x$ as the answer to: "To what power must b be raised to obtain x?"

a. $\left(\frac{1}{3}\right)^x = 27$

$3^{-x} = 3^3$

$-x = 3$

$\boxed{x = -3}$

b. $5^{3x} = 25^{x-1}$

$5^{3x} = 5^{2(x-1)}$

$3x = 2(x-1)$

$3x = 2x - 2$

$\boxed{x = -2}$

c. $4^x = 0.25 \to \frac{1}{4}$

$4^x = 4^{-1}$

$\boxed{x = -1}$

d. $10^x = 15$

$\log_{10} 15 = x$

$\boxed{x = 1.176}$

e. $e^{3x} = 24$

$\ln 24 = 3x$

$3.178 = 3x$

$\boxed{x = 1.059}$

f. $\ln 3x = -0.5003$

$e^{-.5003} = 3x$

Honors: $\boxed{\frac{e^{-.5003}}{3}}$ standard: $\boxed{.202}$

g. $\log_x 64 = \frac{1}{2}$

$\left(x^{\frac{1}{2}}\right)^2 = (64)^2$

$\boxed{x = 4096}$

h. $\log_3 x = 5$

$3^5 = x$

$\boxed{243 = x}$

i. $\log_4 256 = x$

$\frac{\log 256}{\log 4} = x$

$\boxed{x = 4}$

j. $\cancel{\log_7}(2x+5) = \cancel{\log_7}(x-3)$

$2x + 5 = x - 3$

$\boxed{x = -8}$

k. $\log_2(2x^2) = 5$

$2^5 = 2x^2$

$32 = 2x^2$

$16 = x^2$ $\boxed{x = \pm 4}$

l. $\log_{10} x = 2.096910013$

$10^{2.096910013} = x$

$\boxed{x = 125}$

m. $\frac{256e^{2x}}{256} = \frac{1400}{256}$

$e^{2x} = 5.46875$

$\ln 5.46875 = 2x$

$1.699 = 2x$

$\boxed{x = .8495}$

n. $\frac{75}{21} = \frac{21(1.05)^t}{21}$

$3.57 = 1.05^t$

$t = \log_{1.05} 3.57$

$\boxed{t = 26.08}$

o. $10^{x^2+3x-7} = 1{,}000$

$10^{x^2+3x-7} = 10^3$

$x^2 + 3x - 7 = 3$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$\boxed{x = -5 \;\&\; 2}$

Condense (steps) ① #'s out front ② add & sub | Expand (steps) ① mult & divide ② Exponents

Exponential & Logarithmic Functions (continued)

Write the logs in condensed form.	Write the logs in expanded form.
a. $2\log x - x\log y$ $\log x^2 - \log y^x$ ÷ $\boxed{\log \frac{x^2}{y^x}}$	b. $\log x^2 y^3 z^4$ + + $\log x^2 + \log y^3 + \log z^4$ $\boxed{2\log x + 3\log y + 4\log z}$
c. $\log x + 2\log y$ $\log x + \log y^2$ x $\boxed{\log x(y^2)}$	d. $\log(x^2+1)z$ + can't be broken up $\boxed{\log(x^2+1) + \log z}$
e. $\log x + \frac{1}{2}\log y - 2\log z$ $\log x + \log y^{\frac{1}{2}} - \log z^2$ x ÷ $\boxed{\log \frac{xy^{\frac{1}{2}}}{z^2}}$	f. $\log \frac{x^2}{z^6}$ — $\log x^2 - \log z^6$ $\boxed{2\log x - 6\log z}$
g. $\log x + \log y + \log z - 2\log w$ $\log x + \log y + \log z - \log w^2$ x x ÷ $\boxed{\log \frac{xyz}{w^2}}$	h. $\log x^2 y$ + $\log x^2 + \log y$ $\boxed{2\log x + \log y}$

Use the equation given and the properties of logs to solve the problems below:

(1) $A = P\left(1+\frac{r}{n}\right)^{nt}$ (2) $A = Pe^{rt}$	where: • P = original amount deposited or the initial investment • r = the interest rate expressed as a decimal (5% → 0.05) • n = the number of times a year the interest is paid ("quarterly" → means n = 4) • t = the number of years the investment spans

a. Find the value of a $1,000 investment at 6% interest after 10 years compounded:

(a) annually $1000(1+.06)^{10}$

$\boxed{\$1790.85}$

(b) quarterly $1000\left(1+\frac{.06}{4}\right)^{4(10)}$

$\boxed{\$1814.02}$

(c) monthly $1000\left(1+\frac{.06}{12}\right)^{4(12)}$

$\boxed{\$1270.49}$

(d) continuously $1000e^{.06(10)}$

$\boxed{\$1822.12}$

b. If you invest $30,000 at 4.76% interest paid quarterly, how long would it take you to double your money? Round your answer to the nearest hundredth.

$\frac{60,000}{30,000} = \frac{30,000\left(1+\frac{.0476}{4}\right)^{4t}}{30,000}$

$2 = \left(1+\frac{.0476}{4}\right)^{4t}$

$2 = 1.0119^{4t}$

$4t = \log_{1.0119} 2$

$4t = 58.594$

$\boxed{t = 14.6 \text{ years}}$

c. Suppose $2,000 is invested in a 3-year certificate of deposit (CD) that earns 6% interest, compounded continuously. How much will the investment be worth after 3 years?

$2000e^{.06(3)}$

$\boxed{\$2394.43}$

d. You invest $200 at 12.25% earning continuous interest. How many years does it take for your money to increase 5 times its original value? Round your answer to the nearest tenth.

$\frac{1000}{200} = \frac{200e^{.1225t}}{200}$

$5 = e^{.1225t}$

$.1225t = \ln 5$

$\frac{.1225t}{.1225} = \frac{1.609}{.1225}$

$\boxed{t = 13.1 \text{ years}}$