

key

2015—2016 Math III Mock Exam

1. Which expression is equivalent to $(4 - 3i)^2 + (6 + i)^2$?

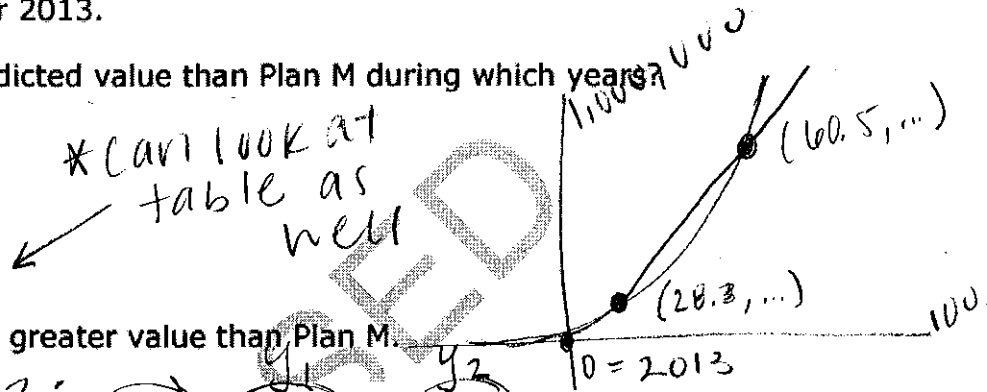
- A 30
 - B 42 - 12i**
 - C 50
 - D 62 - 12i
- $(4 - 3i)(4 - 3i) + (6 + i)(6 + i)$
 $(16 - 12i - 12i + 9i^2) + (36 + 6i + 6i + i^2)$
 $16 - 24i + 9(-1) + 36 + 12i + (-1)$
 $16 - 24i - 9 + 36 + 12i - 1 = 42 - 12i$

2. Samantha invested \$10,000 in each of two different financial plans in 2013. The predicted value of each plan is modeled below.

- Plan M: a rate of 7.5%, compounded continuously. $10,000e^{.075t}$
- Plan N: The value is determined by the function $y = 5x^3 - 50x^2 + 4x + 10,000$, where x is the number of years after 2013.

Plan N has a greater predicted value than Plan M during which years?

- A from 2014 to 2041
- B from 2028 to 2055
- C from 2042 to 2073**
- D Plan N never has a greater value than Plan M.



43%
A.P.E.T. 2

3. What is the solution to the equation $\frac{2x - 3}{x - 1} \times \frac{8x + 1}{4x + 5}$?

- A $-\frac{14}{5}$
 - B $-\frac{14}{9}$
 - C $\frac{14}{9}$
 - D $\frac{14}{5}$**
- method 2: $\frac{2x - 3}{x - 1} \times \frac{8x + 1}{4x + 5}$
 method 1: $(2x - 3)(4x + 5) = (x - 1)(8x + 1)$
 $8x^2 + 10x - 12x - 15 = 8x^2 + x - 8x - 1$
 $8x^2 - 2x - 15 = 8x^2 - 7x - 1$
 $-2x - 15 = -7x - 1$
 $+7x + 15 = +7x + 15$
 $5x = 14$
 $x = 14/5$

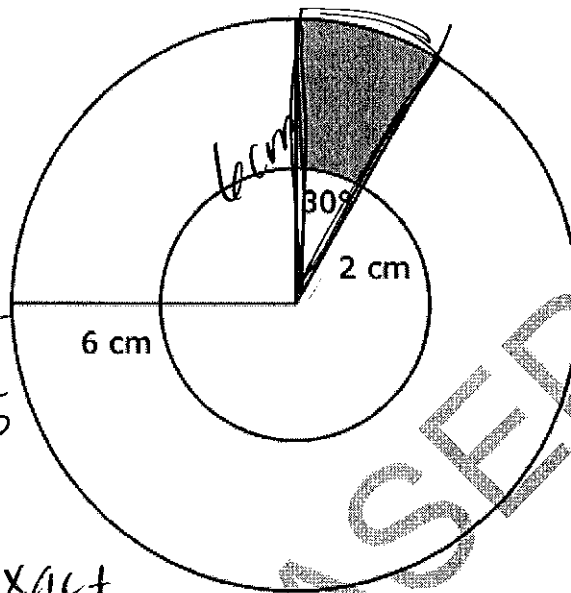
4. In the figure below, the larger circle has a radius of 6 cm, and the smaller circle has a radius of 2 cm.

$$A = \pi r^2$$

$$\left(\frac{\theta}{360}\right) \pi r^2$$

$$\left(\frac{30}{360}\right) \pi (6)^2 = 9.42$$

$$\left(\frac{30}{360}\right) \pi (2)^2 = 1.05$$



$$9.42 - 1.05$$

not exact (about)

What is the approximate area of the shaded region?

A 2.1 cm²

B 3.4 cm²

C 4.2 cm²

D 8.4 cm²

5. Which choice shows the solutions to the equation $8x^2 + 3x = -7$?

A $\frac{-3 \pm \sqrt{215}}{16}$

B $\frac{3 \pm \sqrt{215}}{16}$ (no neg)

C $\frac{-3 \pm \sqrt{233}}{16}$

D $\frac{3 \pm \sqrt{233}}{16}$ (no neg)

$$8x^2 + 3x + 7 = 0$$

$$\frac{-3 \pm \sqrt{(3)^2 - 4(8)(7)}}{2(8)}$$

$$= \frac{-3 \pm \sqrt{9 - 224}}{16} = \frac{-3 \pm \sqrt{-215}}{16}$$

$$= \frac{-3 \pm i\sqrt{215}}{16}$$

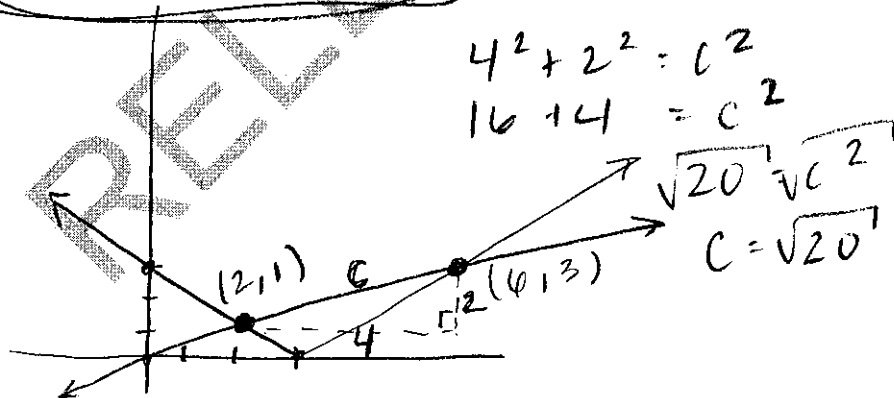
6. A system of equations is shown below.

$$y = |x - 3|$$

$$y = \frac{1}{2}x$$

What is the distance between the points of intersection of the system?

- A $\sqrt{6}$
- B $\sqrt{20}$**
- C $\sqrt{48}$
- D $\sqrt{80}$



4 step problem

Fred drives an average of 15,000 miles per year, and his car gets 20 miles per gallon of gasoline.

- The average cost of gasoline is \$3.25 per gallon.
- He buys a new car.
- In his new car, Fred continues to average 15,000 miles per year, and the average cost of gasoline remains the same.

Approximately how many more miles per gallon does the new car get if Fred has a savings of \$650 per year on gasoline?

- A 5.8 mpg
 - B 7.3 mpg**
 - C 8.8 mpg
 - D 10.3 mpg
- Handwritten work for question 7:
 Old: $\frac{20 \text{ mi}}{1 \text{ gal}} \cdot 15,000 \text{ mi} = 2437.50$
 New: $\frac{15,000 \text{ mi}}{x \text{ gal}} = 1787.50$
 $2437.50 - 1787.50 = 650$
 $1787.50 = 3.25x$
 $x = 550 \text{ gal}$

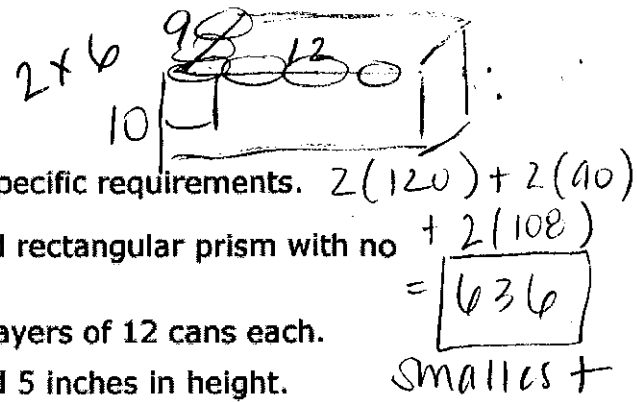
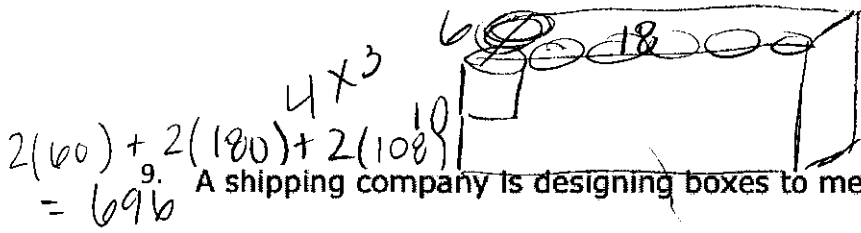
4 new-old 27.2 - 20

8. What value of h is needed to complete the square for the equation $x^2 + 10x - 8 = (x - h)^2 - 33$?

- A -25
- B -5**
- C 5
- D 25

vertex form
 method 1:
 $x^2 + 10x + 25 = 8$
 $(\frac{10}{2})^2 = (5)^2 = 25$
 $(x + 5)^2$
 opposite

method 2:
 graph $x^2 + 10x - 8$
 Find minimum (vertex)
 don't need to comp. the \square
 graph showing vertex at $(-5, -33)$ with "h" indicated.

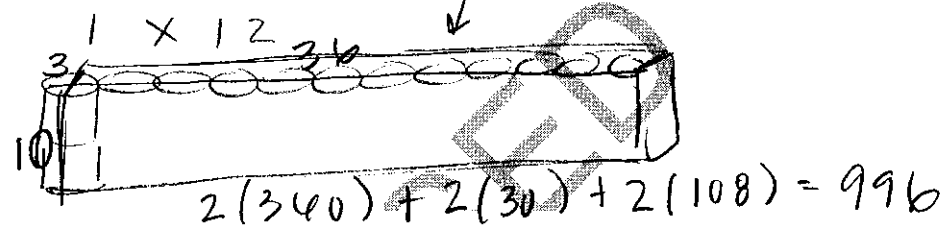


A shipping company is designing boxes to meet specific requirements.

- Each box must be a completely closed rectangular prism with no overlapping material.
- The boxes must hold 24 cans in two layers of 12 cans each.
- The cans are 3 inches in diameter and 5 inches in height.

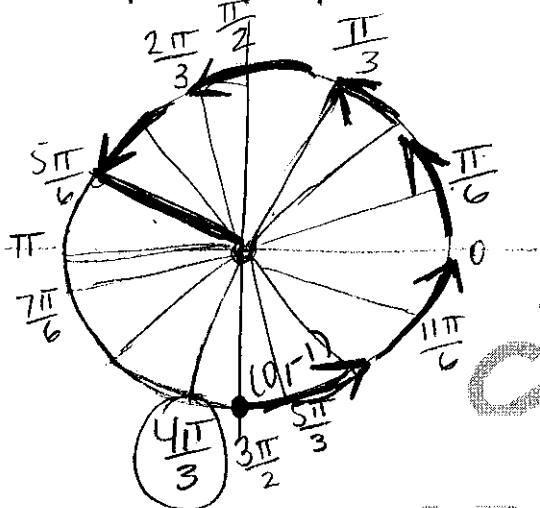
What is the smallest amount of cardboard needed to meet the specifications?

- A 1,080 in.² one layer: 1 x 12 4 x 3 2 x 6
- B 840 in.²
- C 636 in.²
- D 540 in.²



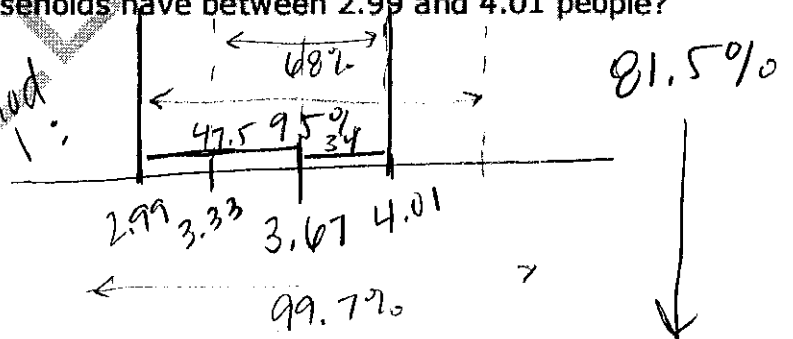
10. William put the tip of his pencil on the outer edge of a graph of the unit circle at the point (0, -1). He moved his pencil tip through an angle of $\frac{4\pi}{3}$ radians in the counterclockwise direction along the edge of the circle. At what angle of the unit circle did William's pencil tip stop?

- A $\frac{\pi}{3}$
- B $\frac{5\pi}{6}$
- C $\frac{7\pi}{6}$
- D $\frac{5\pi}{3}$



11. A town has 685 households. The number of people per household is normally distributed with a mean, μ , of 3.67 and a standard deviation, σ , of 0.34. **Approximately** how many households have between 2.99 and 4.01 people?

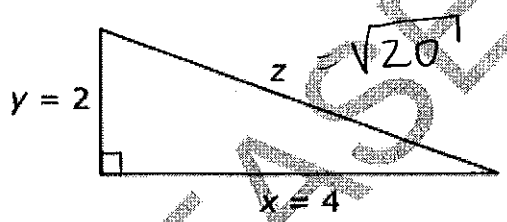
- A 493 households
- B 520 households
- C 558 households
- D 575 households



Method 2:
 $\text{normalcdf}(2.99, 4.01, 3.67, .34) = .818$
 $685 \times .818 = 560$

$685 \times .815 = 558.3$

15. A right triangle is shown below.



$$2^2 + 4^2 = z^2$$

$$4 + 16 = z^2$$

$$\sqrt{20} = \sqrt{z^2}$$

$$z = \sqrt{20}$$

Which expression would result in an irrational number?

- A $x^2 + y^2$ $4^2 + 2^2 = 16 + 4 = 20$ R
- B $\frac{1}{2}xy$ $\frac{1}{2}(4)(2) = 2$ R
- C** $x + y + z$ $4 + 2 + \sqrt{20} = 6 + \sqrt{20}$ **I**
- D $x^2 - z^2$ $(4)^2 - (\sqrt{20})^2 = 16 - 20 = -4$ R

41%
A.P.E.T. 11

16. Let $f(x) = 14x^3 + 28x^2 - 46x$ and $g(x) = 2x + 7$. Which is the solution set to the equation $\frac{1}{12}f(x) = g(x)$?

- A $\{-3, 0, 1\}$ y_1 y_2
- B** $\{-3, -1, 2\}$
- C $\{-2, 1, 3\}$
- D $\{1, 5, 11\}$ X

Looking for X-values!
Y-values!

$$y_1 = \left(\frac{1}{12}\right)(14x^3 + 28x^2 - 46x)$$

$$y_2 = 2x + 7$$

51%
A.SSE.4

17. A board is made up of 9 squares. A certain number of pennies is placed in each square, following a geometric sequence. The first square has 1 penny, the second has 2 pennies, the third has 4 pennies, etc. When every square is filled, how many pennies will be used in total?

Method 1:

1	2	4
8	16	32
64	128	256

last amount

$$1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 = 511$$

Method 2:

$$a_1 = 1 \quad r = 2$$

$$n = 9$$

$$S_n = \frac{1(1 - 2^9)}{1 - 2} = \frac{1 - 512}{-1} = \frac{-511}{-1} = 511$$

30%
A.PE.1.4

18. The equation $2x^2 - 5x = -12$ is rewritten in the form of $2(x - p)^2 + q = 0$. What is the value of q ?

A $\frac{167}{16}$

B $\frac{71}{8}$

C $\frac{25}{8}$

D $\frac{25}{16}$

method 1: graph
 $2x^2 - 5x + 12 = 0$

$(\frac{-5}{2})^2 = (\frac{-5}{4})^2 = \frac{25}{16}$

add to both sides

method 2:
complete the square

$(2)(x^2 - \frac{5}{2}x + \frac{25}{16}) = -12$
careful! $\nearrow +\frac{25}{8}$

$(x - \frac{5}{4})^2 = -\frac{71}{8}$

$(x - \frac{5}{4})^2 + \frac{71}{8} = 0$

22%
F.L.E. 3

19. Which function goes to positive ∞ most quickly as x increases?

A $y = \log(x) + 100$

B $y = e^{x-9} - 3$

C $y = x^2 + 5x + 6$

D $y = 3x^5 + 4x^3 - 11x - 6$

* can graph all & look at table to see which ones has highest y-values

* exponential is always the answer

15%
F.T.F. 1

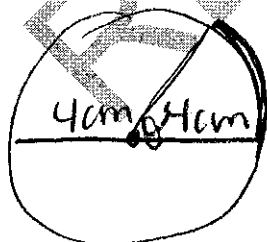
20. The diameter of a circle is 8 centimeters. A central angle of the circle intercepts an arc of 12 centimeters. What is the radian measure of the angle?

A $\frac{3}{2}$

B 3

C 4

D 8π



12cm

$(\frac{\theta}{2\pi})2\pi r$

$C = 2\pi r$

or

$\frac{12}{4} = \frac{4\theta}{4}$

$\theta = 3$

21. Suppose $p(x) = x^3 - 2x^2 + 13x + k$. The remainder of the division of $p(x)$ by $(x + 1)$ is -8 . What is the remainder of the division of $p(x)$ by $(x - 1)$?

A -8 $(-1)^3 - 2(-1)^2 + 13(-1) + k = -8$

~~B 8~~ $-1 - 2 - 13 + k = -8$

C 16 $-16 + k = -8$

D 20 $+16 + k = -8$

$k = 8$

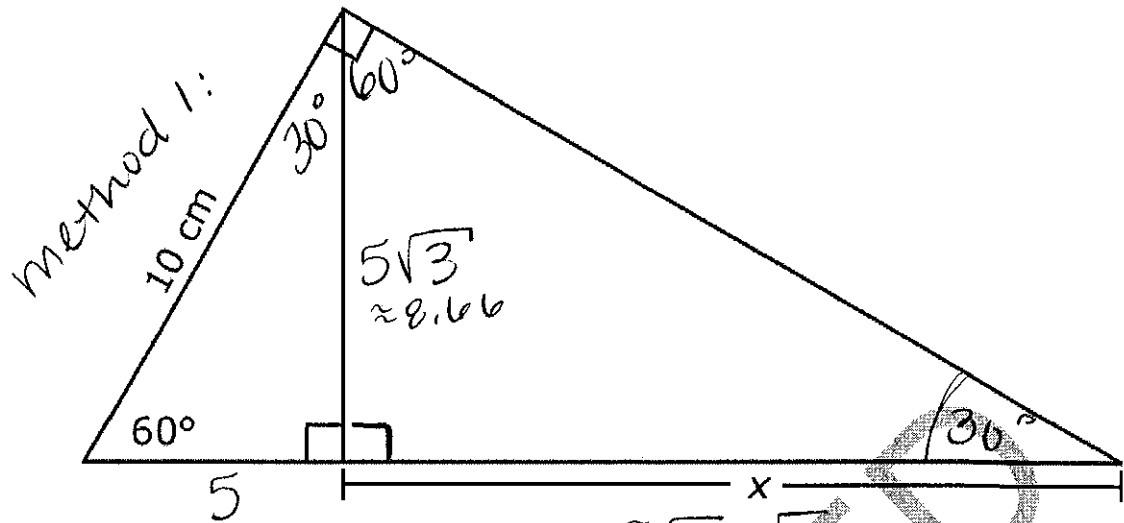
$(1)^3 - 2(1)^2 + 13(1) + 8$

$1 - 2 + 13 + 8 = \boxed{20}$

* can always use synthetic division

61%
4.5 P.T. 5

22. What is the value of x in the triangle below?



A $\frac{5\sqrt{3}}{2}$ cm

B $5\sqrt{3}$ cm

C 10 cm

D 15 cm

method 1:
method 2: $\sin 60 = \frac{5\sqrt{3} \cdot \sqrt{3}}{10} = \frac{15}{10}$
 $10 \sin 60 = x = 8.66$
 $\tan 30 = \frac{8.66}{x}$
 $x = \frac{8.66}{\tan 30} \approx 15$

38%
4. M.K. 3

23. To completely cover a spherical ball, a ball company uses a total area of 36 square inches of material. What is the maximum volume the ball can have?

(Note: Surface area of a sphere = $4\pi r^2$. Volume of a sphere = $\frac{4}{3}\pi r^3$.)

A 27π cubic inches

B $36\sqrt{\pi}$ cubic inches

C $\frac{36}{\sqrt{\pi}}$ cubic inches

D $\frac{27}{\pi}$ cubic inches

$4\pi r^2 = 36$
 $\pi r^2 = 9$
 $r^2 = \frac{9}{\pi}$
 $r = \frac{3}{\sqrt{\pi}}$
 $V = \frac{4}{3}\pi \left(\frac{3}{\sqrt{\pi}}\right)^3$
 $= \frac{4}{3}\pi \frac{27}{\sqrt{\pi}\sqrt{\pi}\sqrt{\pi}}$
 $= \frac{4 \cdot 9}{\sqrt{\pi}} = \frac{36}{\sqrt{\pi}}$

36%
4.MG.3

1 acre = X acre
43,560 ft² = 4.5 mill. ft² ← 4,050,000 ft² 900 ft
X ≈ 93 acre

24. A farmer wants to buy between 90 and 100 acres of land. He is interested in a rectangular piece of land that is 1,500 yards long and 300 yards wide. 1,500 yd
\$87,000 300 yd
3 ft = 1 yd

Criteria 1
Criteria 2

If the farmer does not want to spend more than \$900 an acre, does the land meet all of his requirements? (1 acre ≈ 43,560 ft²)

A Yes, the amount of land satisfies his needs, and the price is low enough.
B No, the price is low enough, but there is too much land.
C No, the price is low enough, but there is not enough land.
D No, the amount of land satisfies what he needs, but the price is too high.

$\frac{3 \text{ ft}}{1 \text{ yd}} = \frac{X \text{ ft}}{1,500 \text{ yd}}$
 $X = 4500 \text{ ft}$

$\frac{3 \text{ ft}}{1 \text{ yd}} = \frac{Y \text{ ft}}{300 \text{ yd}}$
 $Y = 900 \text{ ft}$

(2) 93 acre x \$900 = \$83,700
asking \$87,000 ↑ goal

49%
S.ID.4

25. In a set of test scores that are normally distributed, a test score of 76 is 3 standard deviations below the mean. A score of 88 is 1 standard deviation above the mean. What is the mean of the data?

A 79
B 82
C 84
D 85

$\frac{88 + 76}{2} = 82$
 $\frac{88 + 82}{2} = 85$

60%
F.T.F. 1

26. What is the **approximate** length of the arc subtended by an angle of $\frac{4\pi}{3}$ radians on a circle with a radius of 6.00 meters?

A 12.57 meters
B 14.14 meters
C 25.13 meters
D 28.27 meters

$C = 2\pi r$
 $(\frac{\theta}{2\pi}) 2\pi r$
 $(\frac{4\pi}{3}) 2\pi (6)$

$\frac{4\pi}{3} \cdot 6 = \frac{24\pi}{3} = 8\pi = 25.13$

100