Name: Key Date: ________

Algebra II, Period ___ Math Department

Final Exam Review Packet - Algebra II

- This review packet contains questions that are similar to the type of problems that you will encounter on the exam.
- The in-class review is not meant to re-teach you everything from the second semester. It will be a quick, but thorough overview of the material.
- It is recommended that you work on this review packet leading up to your exam day so you have questions ready. Don't wait till the last minute.
- Remember that the exam counts for 20% or your course grade.
- Reviewing for the exam is **YOUR** responsibility.
- If you have questions as you prepare, make arrangements to see your teacher.

I. Quadratics Equations

Solve each of the following equations using factoring.

a. $x^2-36=0$

$(x+6)(x-6)=0$

$\boxed{x=-6 \text{ \& } 6}$

b. $7x^2-14x=0$

$7x(x-2)=0$

$7x=0 \quad x-2=0$

$\boxed{x=0 \text{ \& } x=2}$

c. $x^3-6x^2-7x=0$

$x(x^2-6x-7)=0$

$x(x-6)(x+1)=0$

$x=0 \quad x-6=0 \quad x+1=0$

$\boxed{x=0,6,-1}$

d. $6x^2+7x-3=0$

$9,-2 \quad -18$

$6x^2+9x \mid -2x-3=0$

$3x(2x+3)-1(2x+3)=0$

$(3x-1)(2x+3)=0$

$3x-1=0 \quad 2x+3=0$

$\boxed{x=1/3 \text{ \& } -3/2}$

e. $3x^2+3x-36=0$

$3(x^2+x-12)=0$

$3(x+4)(x-3)=0$

$3(x+4)=0 \quad x-3=0$

$\boxed{x=-4,3}$

f. $32x^2-2=0$

$2(16x^2-1)=0$

$2(4x+1)(4x-1)=0$

$2(4x+1)=0 \quad 4x-1=0$

$\boxed{x=\pm 1/4}$

g. $x^3-2x^2 \mid -9x+18=0$

$x^2(x-2) \mid -9(x-2)=0$

$(x^2-9)(x-2)=0$

$x^2-9=0 \quad x-2=0$

$x^2=9 \quad x=2$

$\boxed{x=2 \text{ \& } \pm 3}$

h. $x^3-3x^2 \mid +6x-18=0$

$x^2(x-3) \mid 6(x-3)=0$

$(x^2+6)(x-3)=0$

$x^2+6=0 \quad x-3=0$

$x^2=-6$

$\boxed{x=3 \text{ \& } \pm i\sqrt{6}}$

Quadratics Equations (continued) * not solve!

Factor each polynomial COMPLETELY.

Sum of Two Cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$

Difference of Two Cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$

a. x^3+27

$\boxed{(x+3)(x^2-3x+9)}$

b. $8x^3-125$

$\boxed{(2x-5)(4x^2+10x+25)}$

c. x^4+5x^2-14

$\boxed{(x^2+7)(x^2-2)}$

d. $2x^5-18x^3+40x$

$2x(x^4-9x^2+20)$

$2x(x^2-5)(x^2-4)$

$\boxed{2x(x^2-5)(x+2)(x-2)}$

Solve each of the following equations using the Quadratic Formula.

Quadratic Formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

a. $4x^2+6x+1=0$

$\dfrac{-6\pm\sqrt{6^2-4(4)(1)}}{2(4)}$

$=\dfrac{-6\pm\sqrt{36-16}}{8}=\dfrac{-6\pm\sqrt{20}}{8}$

$20 = 2 \cdot 10$, $10 = 2 \cdot 5$

$=\dfrac{-6\pm2\sqrt{5}}{8}=\boxed{\dfrac{-3\pm\sqrt{5}}{4}}$

b. $x^2+2x+2=0$

$\dfrac{-2\pm\sqrt{2^2-4(1)(2)}}{2(1)}=\dfrac{-2\pm\sqrt{4-8}}{2}$

$=\dfrac{-2\pm\sqrt{-4}}{2}=\dfrac{-2\pm2i}{2}=\boxed{-1\pm i}$

c. $2x^2+3x-5=0$

$\dfrac{-3\pm\sqrt{(3)^2-4(2)(-5)}}{2(2)}$

$=\dfrac{-3\pm\sqrt{9+40}}{2}=\dfrac{-3\pm\sqrt{49}}{2}=\dfrac{-3\pm7}{2}$

$\dfrac{-3+7}{2}=\boxed{2}$ $\quad\dfrac{-3-7}{2}=\boxed{-5}$

d. $3x^2-2x-7=0$

$\dfrac{2\pm\sqrt{(-2)^2-4(3)(-7)}}{2(3)}=\dfrac{2\pm\sqrt{4+84}}{6}$

$\dfrac{2\pm\sqrt{88}}{6}$ $\quad 88 \to 8, 11;\ 8 \to 2, 4;\ 4 \to 2, 2$

$\dfrac{2\pm2\sqrt{22}}{6}=\boxed{\dfrac{1\pm\sqrt{22}}{3}}$

II. Powers, Roots, and Radicals

Rewrite the expression with positive exponents. Evaluate where possible.

a. $(-3)^{-4}$ neg means to flip

$\frac{1}{-3^4}$

$= \boxed{\frac{1}{81}}$

b. $\frac{4}{x^0+7}$ 1 $= \frac{4}{1+7} = \frac{4}{8}$

$= \boxed{\frac{1}{2}}$

c. $3x^3(2x)^2$

$3x^3(4x^2)$

$\boxed{12x^5}$

d. $\frac{8a^4b^6}{2(a^5b)^2}$

$\frac{8a^4b^6}{2a^{10}b^2} = \boxed{\frac{4b^4}{a^6}}$

e. $4(x^{-3}y^4)(-3xy^2)^2$

$\frac{4x^{-3}y^4 9x^2y^4}{1}$

$\frac{36x^2y^6}{x^3} = \boxed{\frac{36y^6}{x}}$

f. $\frac{\cancel{20}\,5(a^{-4}b^{-2})}{\cancel{8}\,2(a^{-2}b^4)^{-2}}$

$\frac{5a^{-4}b^{-2}}{a^4b^{-8}}$

$\frac{5b^8}{a^4a^4b^2} = \boxed{\frac{5b^6}{a^8}}$

Solve the radical or rational exponent equation.

a. $\left(x^{\frac{1}{5}}\right)^5 = (2)^5$

$\boxed{x=32}$

b. $2\sqrt{3x-1}+3=11$

$-3 \quad -3$

$\frac{2\sqrt{3x-1}}{2} = \frac{8}{2}$

$(\sqrt{3x-1})^2 = 4^2$

$3x-1=16$

$3x=17$ $\boxed{x=\frac{17}{3}}$

c. $4x^2=64$

$\frac{4x^2}{4} = \frac{64}{4}$

$\sqrt{x^2}=\sqrt{16}$

$\boxed{x=\pm 4}$

d. $2(x-2)^{\frac{1}{4}}-3=159$

$+3 \quad +3$

$\frac{2(x-2)^{1/4}}{2} = \frac{162}{2}$

$\left((x-2)^{1/4}\right)^4 = (81)^4$

$x-2 = 43{,}046{,}721$

$+2 \quad +2$

$\boxed{x = 43{,}046{,}723}$

e. $(\sqrt{2x+4})^2 = (\sqrt{x+2})^2$

$2x+4 = x+2$

$-x \; -4 \quad -x \; -4$

$\boxed{x=-2}$

f. $\sqrt[3]{x}-6=-2$

$+6 \quad +6$

$(\sqrt[3]{x})^3 = 4^3$

$\boxed{x=64}$

III. Simplifying Rational Expressions

Simplify the Rational Expression using Multiplication or Division.

a. $\dfrac{x^2+4x-12}{x^2(x^2+9x+18)}\cdot\dfrac{6x^2}{1}$

$\dfrac{(x+\cancel{6})(x-2)}{\cancel{x^2}(x+\cancel{6})(x+3)}\cdot\dfrac{6\cancel{x^2}}{1}=\boxed{\dfrac{6(x-2)}{(x+3)}}$

b. $\dfrac{3x^2-12}{5x-10}\cdot\dfrac{1}{2x+4}$

$3(x^2-4)$

$\dfrac{3(x+\cancel{2})(x-\cancel{2})}{5(x-\cancel{2})}\cdot\dfrac{1}{2(x+2)}$

$\boxed{\dfrac{3}{10}}$

c. $\dfrac{x^2-4}{x^2+4}\cdot\dfrac{x+2}{x-2}$ $\quad\dfrac{(x+2)(\cancel{x-2})}{x^2+4}\cdot\dfrac{(x+2)}{(\cancel{x-2})}$

$\boxed{\dfrac{x^2+4x+4}{x^2+4}}$

d. $\dfrac{5x^2-20}{25x^2}\cdot\dfrac{x}{x-2}$

$5(x^2-4)$

$\dfrac{\cancel{5}(x+2)(\cancel{x-2})}{5\,\cancel{25}\,x\cancel{^2}}\cdot\dfrac{\cancel{x}}{\cancel{x-2}}$

$\boxed{\dfrac{x+2}{5x}}$

e. $\dfrac{x^2+x-30}{1}\cdot\dfrac{x}{x^2+6x}$

$\dfrac{(x+\cancel{6})(x-5)}{1}\cdot\dfrac{\cancel{x}}{\cancel{x}(x+\cancel{6})}$

$\boxed{x-5}$

f. $\dfrac{\overset{2}{\cancel{12}}\cancel{x}\cancel{y^3}\overset{1}{\cancel{z}}}{\underset{1}{\cancel{6}}x^{\cancel{3}1}\cancel{y^2}z^{\cancel{2}1}}$

$\boxed{\dfrac{2y}{xz}}$

g. $\dfrac{x^3+3x^2}{2x}\div\dfrac{x^2+5x+6}{5x^3}$

$\dfrac{x^2(\cancel{x+3})}{2\cancel{x}}\cdot\dfrac{5x^{\cancel{3}2}}{(\cancel{x+3})(x+2)}$

$\boxed{\dfrac{5x^4}{2(x+2)}}$

h. $\dfrac{x^2+x-20}{x+1}\div\dfrac{11x+55}{x+1}$

$\dfrac{(x+\cancel{5})(x-4)}{(\cancel{x+1})}\cdot\dfrac{(\cancel{x+1})}{11(\cancel{x+5})}$

$\boxed{\dfrac{x-4}{11}}$

i. $\dfrac{x^2+5x+6}{x+3}\div\dfrac{x^2-4}{x+1}$

$\dfrac{(\cancel{x+3})(\cancel{x+2})}{(\cancel{x+3})}\cdot\dfrac{(x+1)}{(\cancel{x+2})(x-2)}$

$\boxed{\dfrac{x+1}{x-2}}$

j. $\dfrac{x^2+6x-7}{3x^2}\div\dfrac{x+7}{6x}$

$\dfrac{(\cancel{x+7})(x-1)}{\cancel{3}x\cancel{^2}}\cdot\dfrac{2\,\cancel{6x}}{(\cancel{x+7})}$

$\boxed{\dfrac{2(x-1)}{x}}$

Simplifying Rational Expressions (continued)

Simplify the Rational Expression using Addition or Subtraction. (LCD = ?)

a. $\frac{5}{5}\frac{4}{3x^2}+\frac{2}{5x}\frac{3x}{3x}$

$\frac{20}{15x^2}+\frac{6x}{15x^2}=\boxed{\frac{20+6x}{15x^2}}$

---

b. $\frac{3}{2x-2}+\frac{x+1}{4}$

$\frac{2}{2}\frac{3}{2(x-1)}+\frac{x+1}{4}\frac{(x-1)}{(x-1)}=\frac{6}{4(x-1)}+\frac{x^2-1}{4(x-1)}=\boxed{\frac{x^2-5}{4(x-1)}}$

$\left(\frac{2x+1}{2x+1}\right)\frac{4}{3x^3}+\frac{x}{6x^3+3x^2}\rightarrow\frac{x}{3x^2(2x+1)}\left(\frac{x}{x}\right)$

$\frac{8x+4}{3x^3(2x+1)}+\frac{x^2}{3x^3(2x+1)}=\boxed{\frac{x^2+8x+4}{3x^3(2x+1)}}$

---

d. $\frac{5x-1}{x^2+2x-8}-\frac{6}{x+4}$

foil!

-8, 4, -2, 2

$\frac{5x-1}{(x+4)(x-2)}-\frac{6}{(x+4)}\frac{(x-2)}{(x-2)}=\frac{5x-1-6x+12}{(x+4)(x-2)}=\boxed{\frac{-x+11}{(x+4)(x-2)}}$

---

e. $\frac{x+1}{x^2+6x+9}-\frac{1}{x^2-9}$

$\frac{(x-3)}{(x-3)}\frac{x+1}{(x+3)(x+3)}-\frac{1}{(x+3)(x-3)}\frac{(x+3)}{(x+3)}=$

$\frac{x^2+1x-3x-3}{(x-3)(x+3)(x+3)}-\frac{x-3}{(x-3)(x+3)(x+3)}$

$\boxed{\frac{x^2-3x-6}{(x+3)(x+3)(x-3)}}$

IV. Solving Rational Equations

Solve each rational equation.

a. $\frac{3}{x+4} = \frac{9}{x-2}$

$3x - 6 = 9x + 36$
$-3x \quad -3x$
$-6 = 6x + 36$
$-36 \quad -36$
$\frac{-42}{6} = \frac{6x}{6}$
$\boxed{x = -7}$

b. $\frac{4x}{x-1} = \frac{x}{x^2-1}$

$4x^3 - 4x \quad x^2 - x$
$4x^3 - x^2 - 3x = 0$
$x(4x^2 - x - 3) = 0$
$-12 \quad -4, 3$
$x(4x^2 - 4x + 3x - 3)$
$4x(x-1) \; 3(x-1)$
$x(4x+3)(x-1) = 0$
$x = 0 \quad 4x + 3 = 0 \quad x - 1 = 0$
$\boxed{x = 0, -\frac{3}{4}}$

Work is correct but $x \neq 1$

c. $\frac{3}{x^2-4} = \frac{2}{x+2} + \frac{x}{x-2}$

$\frac{3}{(x+2)(x-2)} = \frac{2}{(x+2)}\frac{(x-2)}{(x-2)} + \frac{x}{(x-2)}\frac{(x+2)}{(x+2)}$

$3 = 2x - 4 + x^2 + 2x \longrightarrow 0 = x^2 + 4x - 7$

$\frac{-4 \pm \sqrt{4^2 - 4(1)(-7)}}{2(1)} = \frac{-4 \pm \sqrt{44}}{2} = \frac{-4 \pm 2\sqrt{11}}{2} =$
$\boxed{-2 \pm \sqrt{11}}$

d. $\frac{3x-2}{x-2} = \frac{6}{x^2-4} + 1$

$\frac{3x-2}{x-2}\frac{(x+2)}{(x+2)} = \frac{6}{(x+2)(x-2)} + \frac{1}{1}\frac{(x+2)(x-2)}{(x+2)(x-2)}$

$3x^2 + 6x - 2x - 4 = 6 + x^2 - 4$
$-x^2 \quad -2 \quad -x^2$

$2x^2 + 4x - 6 = 0$
$(2x^2 + 6x) - (2x - 6) = 0$
$-12 \quad 6, -2 \quad 4$
$2x(x+3) - 2(x+3)$
$(2x - 2)(x+3)$
$\frac{2x}{2} = \frac{2}{2}$
$\boxed{x = -3}$
$\boxed{x = 1}$

e. $\frac{x}{x+2} = \frac{3x+1}{x-1} + \frac{4}{x^2+x-2}$

$12 \quad 6, 2 \quad 8$

$\frac{x}{x+2}\frac{(x-1)}{(x-1)} = \frac{3x+1}{x-1}\frac{(x+2)}{(x+2)} + \frac{4}{x^2+x-2}$

$x^2 - x = 3x^2 + 6x + x + 2 + 4$
$x^2 - x = 3x^2 + 7x + 6$
$-x^2 + x \quad -x^2 \quad +x$

$0 = 2x^2 + 8x + 6$
$(2x^2 + 6x) + (2x + 6)$
$2x(x+3) + 2(x+3)$
$\boxed{x = 1 \quad x = -3}$

VI. Inverses

Find the inverse of each function.

a. $f(x)=2x+5$

$x = 2y + \cancel{5}$
$-5 \quad -\cancel{5}$
$x-5=2y$
$\boxed{y^{-1} = \frac{x-5}{2}}$

b. $f(x)=\sqrt[3]{2x+4}$

$x = \sqrt[3]{2y+4}$
$x^3 = 2y+4$
$x^3 - 4 = 2y$
$\boxed{y^{-1} = \frac{x^3-4}{2}}$

c. $f(x)=5-\frac{5}{2}x$

$\boxed{y^{-1} = \frac{-2x+1}{5}}$
$x = 5 - \frac{5}{2}y$
$-\frac{2}{5}(x-5) = \left(-\frac{5}{2}y\right)\left(-\frac{2}{5}\right)$

d. $f(x)=\frac{x-2}{4}$

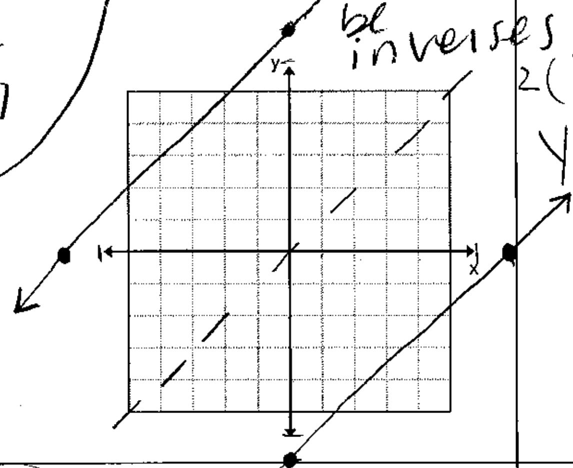
$x = \frac{y-2}{4}$
$4x = y-2$
$y^{-1} = \frac{4x}{2} = \boxed{2x}$

Verify that the two functions are inverses of each other using composite functions. Then, verify (a) and (b) by graphing.

a. $f(x)=x+7$, $g(x)=x-7$

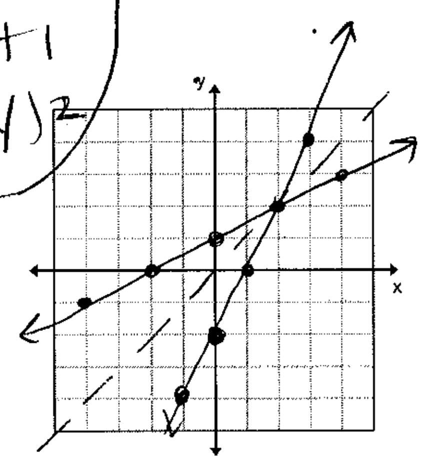
$x = y + \cancel{7}$
$-7 \quad -\cancel{7}$
$y^{-1} = x-7$
$\boxed{\text{Yes!}}$

*must reflect over y=x to be inverses



b. $f(x)=\frac{1}{2}x+1$, $g(x)=2x-2$

$x = \frac{1}{2}y+1$
$2(x-1) = \left(\frac{1}{2}y\right)2$
$y^{-1} = 2x-2$
$\boxed{\text{Yes!}}$



c. $f(x)=\frac{1}{3}x^2$, $g(x)=\sqrt{3x}$

$x = \frac{1}{3}y^2$
$3x = y^2$
$y^{-1} = \pm\sqrt{3x}$
$\boxed{\text{NO!}}$

*must be ±!

d. $f(x)=\frac{x^5+2}{7}$, $g(x)=\sqrt[5]{7x-2}$

$x = \frac{y^5+2}{7}$
$7x = y^5+2$
$7x-2 = y^5$
$\sqrt[5]{7x-2} = y^{-1}$
$\boxed{\text{Yes!}}$

The graph of the inverse function is the reflection of the original function over what line?

$\boxed{y = x}$

Divide $\dfrac{3h^3-4h^2+4}{h^2-2h+2}$ "no "h""

$$
\begin{array}{r}
3h+2 \\
h^2-2h+2 \overline{\big)\, 3h^3-4h^2+0h+4} \\
-(3h^3+6h^2+6h) \downarrow \\
\hline
2h^2-6h+4 \\
-(2h^2+4h+4) \\
\hline
-2h
\end{array}
$$

$$\boxed{3h+2 - \frac{2h}{h^2-2h+2}}$$

Solve: $3x^3-17x^2+15x-25$ ← can't be factored ∴

graph! solution at $x=5$

$$
\begin{array}{c|cccc}
 & 3 & 2 & 1 & 0 \\
5 & 3 & -17 & 15 & -25 \\
 & \downarrow & 15 & -10 & 25 \\
\hline
 & 3^{2} & -2^{1} & 5^{0} & 0
\end{array}
$$

$$3x^2-2x+5=0$$

$$\frac{2 \pm \sqrt{(-2)^2-4(3)(5)}}{2(3)} = \frac{2 \pm \sqrt{4-60}}{6} = \frac{2 \pm \sqrt{-56}}{6}$$

$\cdot 7 \quad 2 \quad 2\,2$

$14 \quad 4$

$$= \frac{2 \pm 2i\sqrt{14}}{6} = \boxed{\left\{ \frac{1 \pm i\sqrt{14}}{3} \text{ \& } 5 \right\}}$$

Is $x-5$ a factor of the function $x^3+x^2-27x-15$? Show work supporting your answer

$x=5$

$$f(5) = (5)^3+(5)^2-27(5)-15$$

$$125+25-135-15$$

$$=0 \quad \boxed{\text{Yes!}}$$

*This technique is called "remainder Theorem"